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UH-6121

B. E. - II (Sem. - III) (EC/EL/Chem.) Examination

May/June – 2012

Engg. Maths - III

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. - II (Sem. - III) (EC/EL/Chem.)**

Name of the Subject :
☛ **Engg. Maths - III**

☛ Subject Code No. : **6 1 2 1** ☛ Section No. (1, 2,.....) : **1,2**

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figures on right indicate marks.

Section - I

1 (a) Do as directed : **10**

(i) Express the integral $\int_0^1 \int_{4y}^4 e^{x^2} dxdy$, changing the order of integration.

(ii) Express the integral $\int_0^a \int_0^{\sqrt{a^2-y^2}} \left(x^2+y^2\right) dxdy$ changing into polar co-ordinates.

(iii) Find unit normal to the surface $x^2+y^2+z^2=9$ at point (2,2,1).

(iv) Defint divergence of a vector. Find $div\,\bar{r}$ where $\bar{r} = x\hat{i}+y\hat{j}+z\hat{k}$.

(v) Define an irrotational vector. Give one illustration of it.

(b) Attempt any **three** : **12**

(i) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$.

(ii) Evaluate $\iint r^3 dr d\theta$, over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$.

(iii) Find the volume of the solid, bounded by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$.

(iv) Find the mass of an elliptic plate $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if the density at any point (x,y) on it, is μxy, where μ is a constant.

2 (a) Attempt any **two** : **6**

(i) Show that $\nabla^2\left(r^n\right)=n(n+1)r^{n-2}$.

(ii) A fluid motion is given by $\overline{v}=(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$. Is this motion irrotational ? If so, find the velocity potential.

(iii) Find the directional derivative of the function $f(x,y,z)=xy^2+yx^3$ at the point (2,–1,1) in the direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$.

(b) Attempt any **two** : **8**

(i) Apply Green's theorem to evaluate $\oint_c\left[\left(2x^2-y^2\right)dx+\left(x^2+y^2\right)dy\right]$ where c is the boundary of the area enclosed by the x-axis and upper half of the circle $x^2+y^2=a^2$.

(ii) Apply Stoke's theorem to evaluate $\oint_c\left[(x+y)dx+(2x-z)dy+(y+z)dx\right]$ where c is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6).

(iii) Use Divergence theorem to evaluate $\iint_s \overline{F}\cdot\overline{dS}$, where $\overline{F}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$.

3 (a) Express $f(x)=x$ as a half-range cosine series in $0<x<2$. **4**

(b) Attempt any **two** : **10**

(i) Obtain the Fourier series to represent e^x in the interval $0<x<2\pi$.

(ii) Obtain Fourier series for the function $f(x)$ given by

$$f(x)=\begin{cases}1+\dfrac{2x}{\pi}, -\pi \le x \le 0\\ 1-\dfrac{2x}{\pi},\ 0 \le x \le \pi\end{cases}$$

(iii) If $f(x)=\left(\dfrac{\pi-x}{2}\right)^2$ in the interval $0<x<2\pi$, show that $f(x)=\dfrac{\pi^2}{12}+\sum_{n=1}^{\infty}\dfrac{\cos nx}{n^2}$. Hence deduce $\dfrac{1}{1^2}+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\ldots=\dfrac{\pi^2}{6}$.

Section - II

4 (a) Do as directed : **10**

(i) Define Beta function. Show that it is symmetric in its arguments.

(ii) Obtain value of $\Gamma\left(-\frac{1}{2}\right)$.

(iii) Write one-dimensional wave equation. Write its physically consistent solution also.

(iv) Write Cauchy-Riemann equations in cartesian form.

(v) Write statement of Cauchy Integral theorem.

(b) Attempt any **two** : **6**

(i) Prove that $\int_{-\infty}^{\infty} e^{-k^2x^2}\,dx=\dfrac{\sqrt{\pi}}{k}$.

(ii) Prove that $\int_0^1\left(\log\dfrac{1}{x}\right)^{n-1}dx=\Gamma(n)$, $n>0$.

(iii) Show that $\int_0^1 x^5\left(1-x^3\right)dx=\frac{1}{60}$.

(c) Solve any **two** : **6**

(i) $yzp-xzq=xy$.

(ii) $(mx-ny)p+(nx-lz)q=ly-mx$.

(iii) $\dfrac{y-z}{yx}p+\dfrac{z-x}{zx}q=\dfrac{x-y}{xy}$.

5 Attempt any **two** : **12**

(i) Solve the boundary value problem, $\dfrac{ru}{rt}=c^2\dfrac{r^2u}{rx^2}$, $0<x<l$ with conditions : $\dfrac{ru}{rx}(o,t)=\dfrac{ru}{rx}(l,t)=0$ for $t>0$ and $u(x,0)=x$.

(ii) A tightly stretched string with fixed end points $x=0$ and $x=l$, i initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$ where y_0 is a constant. If it is released from rest, from this position, find the displacement $y(x,t)$.

(iii) Solve $\frac{r^2 u}{r x^2} + \frac{r^2 u}{r y^2} = 0$, which satisfies the conditions, $u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin\frac{n\pi x}{l}$.

6 (a) Define an analytic function. If $f(z)$ is an analytic function of z, then prove that **4**

$$\left(\frac{r^2}{r x^2} + \frac{r^2}{r y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$

(b) Attempt any **two** : **6**

(i) Find the regular function whose imaginary part is $\cos x \cosh y$.

(ii) Show that the image of the hyperbola $x^2 - y^2 = 1$, under the transformation $\omega = \frac{1}{z}$ is the lemniscate $\rho^2 = \cos 2\phi$ where $\omega = \rho e^{i\phi}$.

(iii) Find the bilinear transformation which maps $z = 1, i, -1$ onto the points $\omega = i, 0, -i$ respectively.

(c) Attempt any **two** : **6**

(i) Evaluate $\oint_c \frac{e^{-z}}{z+1} dz$, where c is the circle $|z| = 2$.

(ii) Evaluate $\oint_c \frac{z^2+1}{z(2z+1)} dz$, where c is the circle $|z| = 1$.

(iii) Evaluate $\oint_c \frac{e^z}{z^3} dz$, where c is the circle $|z| = \frac{1}{2}$.